Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 20 January 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02R**

Further Pure Mathematics

Paper 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

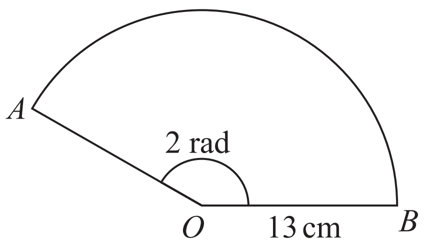

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the sector *AOB* of a circle with centre *O*.
The radius of the circle is 13 cm and angle *AOB* = 2 radians.

(a) Find the length of the arc *AB*.

(1)

(b) Find the area of the sector *AOB*.

(2)

(Total for Question 1 is 3 marks)

2

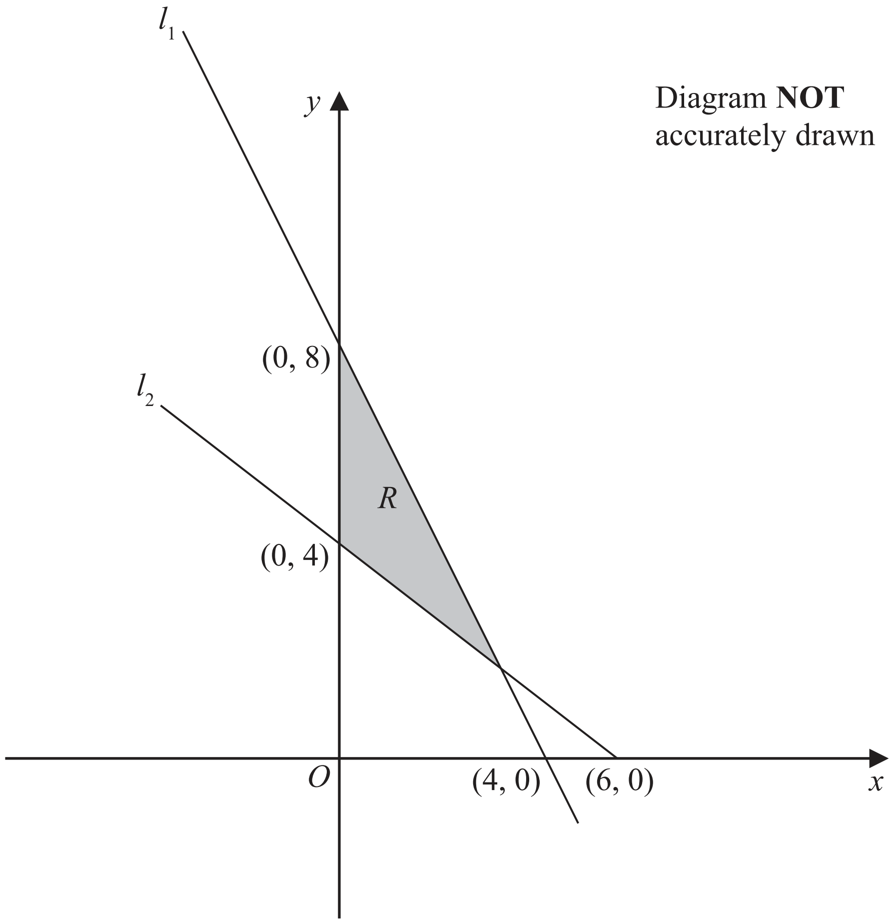

Figure 2

Figure 2 shows the shaded region R bounded by the line l_1, the line l_2 and the y-axis.

The points with coordinates (0, 8) and (4, 0) lie on l_1

The points with coordinates (0, 4) and (6, 0) lie on l_2

(a) Find, in the form $ax + by = c$, where a, b and c integers, an equation of

(i) l_1

(ii) l_2

(3)

(b) Hence write down three inequalities that define the region R.

(3)

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Question 2 continued

(Total for Question 2 is 6 marks)

3 In triangle ABC, $AB = 11$ cm and $BC = 12$ cm.

The area of triangle $ABC = 33\,\text{cm}^2$

Find, in cm to 3 significant figures, the two possible lengths of AC.

(5)

Question 3 continued

(Total for Question 3 is 5 marks)

4

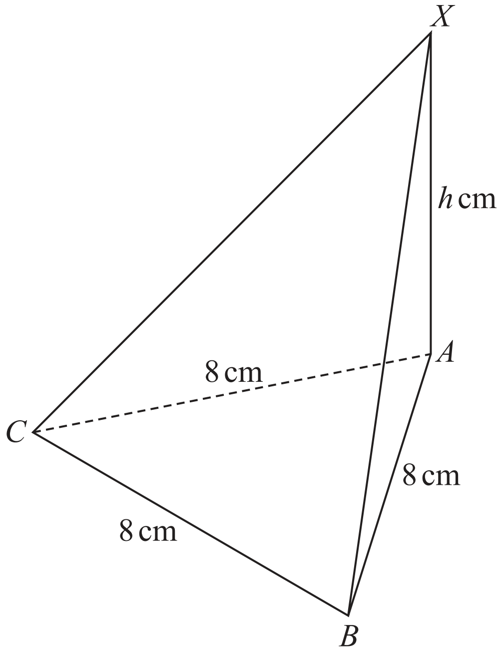

Figure 3

Figure 3 shows a triangular pyramid *ABCX*.
The base *ABC* of the pyramid is an equilateral triangle where $AB = BC = CA = 8$ cm.
The vertex *X* of the pyramid is such that *AX* is perpendicular to the base of the pyramid and $AX = h$ cm.

The volume of the pyramid is $48\sqrt{3}$ cm^3

(a) Show that $h = 9$

(3)

(b) Find, in degrees to one decimal place, the size of angle *BXC*.

(3)

(c) Find, in degrees to one decimal place, the size of the angle between the plane *BCX* and the base *ABC* of the pyramid.

(3)

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Question 4 continued

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Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

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5 (a) Show that $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$

(2)

The roots of the equation $2x^2 + 3x + 6 = 0$ are α and β

Without solving the equation,

(b) find the value of $\alpha^3 + \beta^3$

(2)

(c) Show that $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$

(2)

(d) Form a quadratic equation with integer coefficients that has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$

(6)

Question 5 continued

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Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6

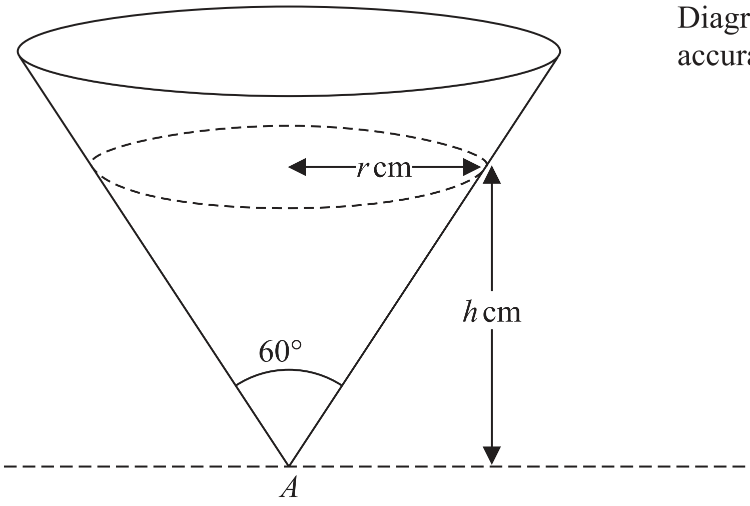

Figure 4

Figure 4 shows a hollow right circular cone fixed with its axis of symmetry vertical.

The cone is inverted and contains liquid, which is dripping out of a small hole at the vertex A of the cone at a constant rate of $0.9\,\text{cm}^3/\text{s}$.

At time t seconds after the liquid starts to drip from the cone, the height of the liquid is h cm above A. The volume of liquid in the cone at time t seconds is $V\,\text{cm}^3$

The vertical angle of the cone is 60°

(a) Show that $V = \frac{1}{9}\pi h^3$

(2)

(b) Find, in cm/s to 3 significant figures, the rate at which the height of the liquid is decreasing when the height of the liquid in the cone above the vertex is 1.2 cm.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 6 marks)

7 The geometric series G has first term a, common ratio r and nth term u_n

Given that $u_4 = e^{x+2}$ and that $u_7 = e^{\frac{2x+1}{2}}$

(a) show that $r = e^{-\frac{1}{2}}$

(3)

(b) Hence find a in terms of e and x.

(3)

Given that the sum to infinity of G can be written as $\dfrac{e^p}{e^{\frac{1}{2}} - 1}$

(c) find an expression for p in terms of x.

(3)

Given that $u_{18} > 1.6$ and that x is an integer,

(d) find the least value of x.

(4)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 13 marks)

8 (a) Write down the value of k such that $\sin 2A = k\sin A\cos A$

(1)

$$g(A) = 2 + 3\cos A - \sin A - 3\sin 2A - 2\cos^2 A$$

Given that $g(A)$ can be written in the form $(p\cos A - \sin A)(q - r\sin A)$ where p, q and r are integers,

(b) find the value of p, the value of q and the value of r.

(3)

(c) Hence solve, in radians to 3 significant figures where appropriate, the equation

$$g(2\theta) = 0 \quad \text{for} \quad 0 \leqslant \theta < \pi$$

(6)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

P 6 0 4 7 6 A 0 2 7 4 0

9 Given that $\dfrac{1}{(2-x)^3}$ can be written as $p(1 - qx)^{-3}$

(a) find the value of p and the value of q.

(2)

(b) Expand $\dfrac{1}{(2-x)^3}$ in ascending powers of x up to and including the term in x^3 and express each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{a+bx}{(2-x)^3} \quad \text{where } a \text{ and } b \text{ are integers}$$

The first three terms of the expansion of $f(x)$ are $\dfrac{3}{8} - \dfrac{43}{16}x + cx^2$

(c) Find the value of a and the value of b.

(3)

(d) Find the exact value of c.

(2)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

10 The equation of a curve C is $y = \mathrm{f}(x)$ where $\mathrm{f}'(x) = 3x^2 - 4x - p$ and $p \neq 0$

The points with coordinates $(2, 0)$ and $(-1, 9)$ lie on C.

(a) Show that C has equation $y = x^3 - 2x^2 - 4x + 8$

(6)

The straight line l has equation $y = 8 - 4x$

(b) Use algebraic integration to find the exact area of the finite region bounded by C and l.

(6)

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 12 marks)

11 The curve C has equation $y = \dfrac{3x-2}{x+1}$

(a) Write down an equation of the asymptote to C which is parallel to the

(i) x-axis (ii) y-axis

(2)

(b) Find the coordinates of the point where C crosses the

(i) x-axis (ii) y-axis

(2)

(c) Sketch C, showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

The straight line l has equation $y = mx + 4$

Given that there are **no** points of intersection between l and C,

(d) show algebraically that the range of possible values of m can be written as

$$a - 2\sqrt{b} < m < a + 2\sqrt{b}$$

where a and b are integers whose values need to be found.

(7)

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Question 11 continued

Question 11 continued

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Question 11 continued

Question 11 continued

(Total for Question 11 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS